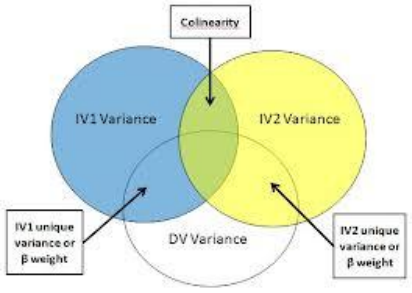
IAFF 6501- 11

Applied Quantitative Analysis

Class 12

Multiple Regression



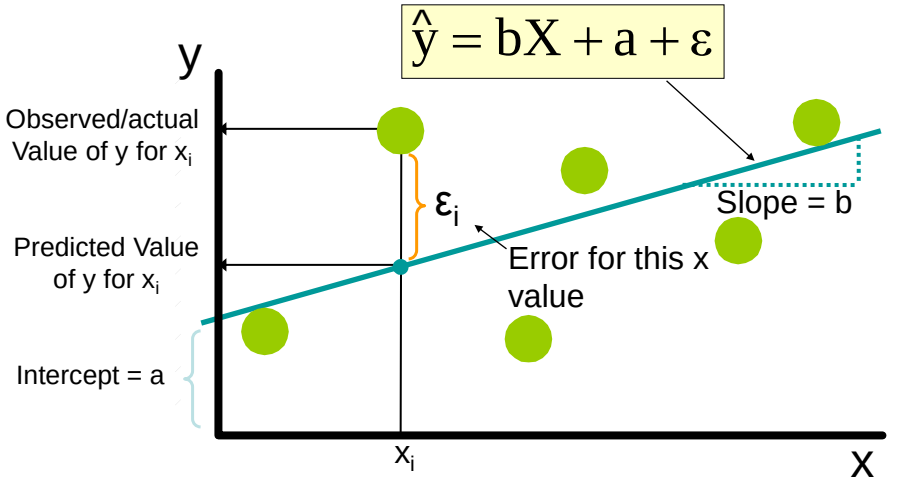
Course Grading Scale

94 -100 A
90-92 A-
87-89 B+
83-86 B
80-82 B-
77-79 C+
73-76 C
70-72 C-
69 and below F

Review
Simple Regression Analysis

- Regression analysis is used to:
  - Predict the value of an outcome variable based on certain values of the predictor variable
  - Extension of correlation analysis

Scatter Plot & Regression Line



Exercise # 1
Linear Regression Analysis in SPSS

- *Is there a relationship between height and shoe size?*
- Dataset in Class 12 folder on BB
- Run linear regression in SPSS
- Graph the results on a scatterplot
- Interpret output

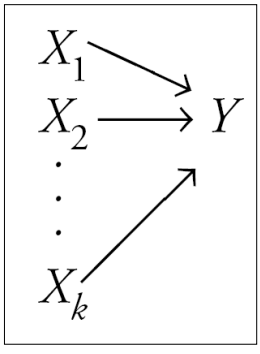
Exercise #2
Income and Education Level

	Education Level (X)	Monthly income (Y) in thousands
1	6 years	1
2	8 years	1.5
3	11 years	1
4	12 years	2
5	12 years	4
6	13 years	2.5
7	14 years	5
8	16 years	6
9	16 years	10
10	21 years	8

1. Enter data into SPSS
2. Run linear regression
3. Graph the results on a scatterplot
4. Interpret results
5. Produce the regression equation
6. Predict the monthly income for 15 years of education

Questions

Multiple Regression



Multiple regression simultaneously considers the influence of *multiple* explanatory variables on a response variable Y

Multiple Regression
2+ Independent Variables

	Education Level (X_1)	Years working (X_2)	Monthly income (Y) in thousands
1	6 years	10	1
2	8 years	14	1.5
3	11 years	8	1
4	12 years	7	2
5	12 years	20	4
6	13 years	15	2.5
7	14 years	17	5
8	16 years	22	6
9	16 years	30	10
10	21 years	10	8

Multiple Regression Model

$$\hat{y} = a + b_1 x_1 + b_2 x_2$$

y = predicted value of the outcome variable

a = y-intercept, where X's equal zero

b = coefficient or slope for each variable

X1 = the value of the first predictor

X2 = the value of the second predictor

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Multiple Regression

1. Test if the two predictors variables in combination are significantly related to, or predictive of the outcome variable, and how much of the variance the predictor variables explain in the outcome variable

2. Test whether the each predictor variable is significantly related to the outcome variable, *controlling for* the other predictor variable

3. Determine which of the predictor variables is the stronger predictor of the outcome variable

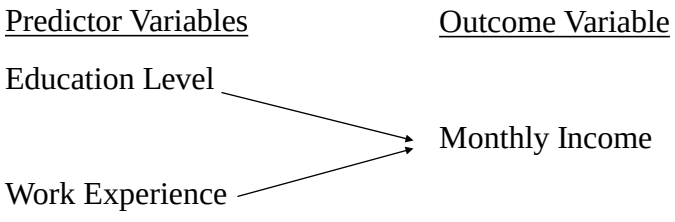
Multicollinearity

- Strong correlations among predictor variables. This makes it difficult to identify the unique relationship between each predictor variable and the outcome variable.

	Education years	Work Exp. Years	Monthly Income
Education yrs	1.00		
Work Exp. yrs	**.31**	1.00	
Monthly income	.826	.695	1.00

Multiple Regression

- We are interested in the relationship between Education and Work Experience and Monthly Income.



Variance Explained

Variance Explained (Model Summary)			
R	R^2	Adjusted *R* Square	Std Error of the Estimate
.946	.896	.866	1.1405

- Multiple correlation coefficient (*R*)
- Coefficient of determination (R^2)
- Adjusted *R* Square: takes into account sample size and number of predictors

Statistical Significance

ANOVA Results

	SS	DF	MS	F	*p*
Regression	78.29	2	39.14	30.09	.000
Residual	9.10	7	1.30		
Total	87.40	9			

Multiple Regression

Intercept *a*

Slope b_1

Slope b_2

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	11.770	1.734		6.787	.000
	Education	-.364	.173	-.412	-2.105	.047
	Work	-.403	.194	-.408	-2.084	.049

a. Dependent Variable: Income

The multiple regression model is:
Monthly Income = 11.8 −.36(X_1) - .40(X_2)

Unstandardized vs. Standardized Coefficients

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	11.770	1.734		6.787	.000
	Education	-.364	.173	-.412	-2.105	.047
	Work	-.403	.194	-.408	-2.084	.049

a. Dependent Variable: Income

- Unstandardized (B): variables on different scales
- Standardized (β): same scale via z-score
- Use standardized coefficients (β) to compare

R does *NOT* equal Causation!!

Questions

Exercise #3 *Multiple Regression*

	Education Level (X_1)	Work Experience in years (X_2)	Monthly income (Y) in thousands
1	6 years	10	1
2	8 years	14	1.5
3	11 years	8	1
4	12 years	7	2
5	12 years	20	4
6	13 years	15	2.5
7	14 years	17	5
8	16 years	22	6
9	16 years	30	10
10	21 years	10	8

1. Add the Work Exp. variable to your dataset
2. Run multiple regression
3. Write regression equation
4. Interpret results
   - Sig, R, R^2, β

CONCLUDING NOTES

- **No class next week**
- **2 Readings in Class 13 Folder**
  - Internal and External Validity
  - Ethics & Research
  - Read for Dec 5th class

See you Dec 5th!